A Practical Method for Calibrating a Coaxial Noise Source with a Waveguide Standard

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***Abstract*—A practical method for calibrating a coaxial noise source with a waveguide standard has been developed by extending the adaptor-changing method proposed by Engen. A practical equation for its noise temperature, the measurement procedure, and the error analysis are described.**

I. INTRODUCTION

MICROWAVE NOISE sources are required for quantitative measurement of receiver power and receiving systems in radio astronomy, earth–space communication, and remote sensing.

A primary standard would preferably be a thermal noise source of the waveguide type rather than the coaxial type from the simplicity of its configuration and the ease of evaluation of its noise temperature. Meanwhile, those of the coaxial type are widely used as the secondary standard since they are small and lightweight. Therefore, the case will arise where the coaxial noise source should be calibrated as to noise temperature with the waveguide standard by placing a coax-waveguide adaptor in front of its output port, as shown in Fig. 1. In this case, the following relationship exists between the noise temperatures of the coaxial noise source and the source at the output port of the adaptor, T_x and T_{xw}:

$$T_{xw} = \alpha T_x + (1-\alpha) T_a \tag{1}$$

with

$$\alpha = \frac{|S_{21}|^2 (1-|\Gamma_x|^2)}{|1-S_{11}\Gamma_x|^2 (1-|\Gamma_{xw}|^2)} \tag{2}$$

where α is the ratio of available power at the adaptor input to that at the output, i.e., the absorption loss in the adaptor; T_a is the ambient temperature of the adaptor; and S_{ij} is the $i-j$ element of the S matrix. Then, in order to determine T_x with this relationship, α must be accurately measured together with T_{xw} and T_a. However, it is very hard to precisely determine its magnitude by measuring its S parameters and the reflection coefficient of the source. On the other hand, a method to avoid the direct measurement of them was proposed by Engen which can provide an easy measurement of its temperature, i.e., the adaptor-changing method [1]. However his expression for noise temperature would be impractical because the radiometer noise temperature is not taken into account in the derivation of it.

By extending Engen's idea, we have developed a practical method providing easy determining of its temperature.

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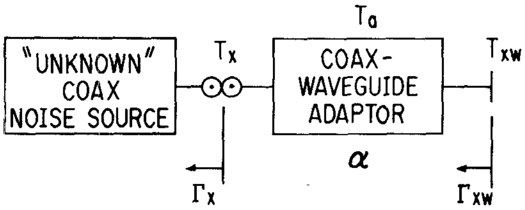

Fig. 1. Coaxial noise source whose output port is changed to the waveguide system with a coax–waveguide adaptor.

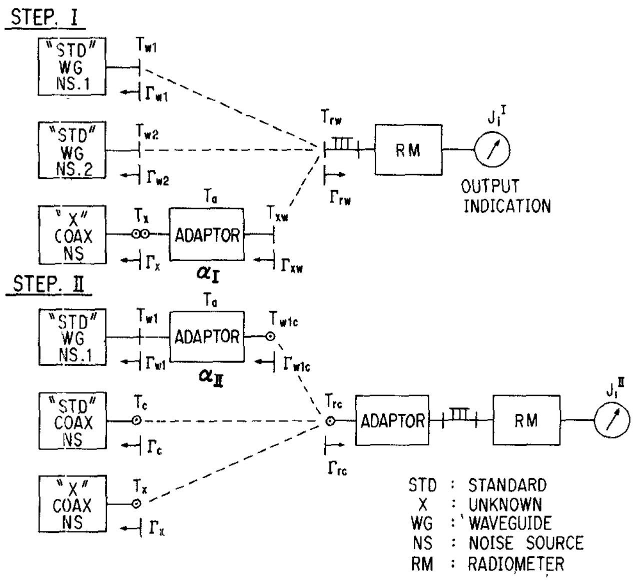

Fig. 2. Schematic diagram depicting the measurement procedure and system for a coaxial connector system of the sexless type.

II. THEORY

A schematic diagram depicting the measurement procedure and system is shown in Fig. 2 with the symbols representing the relevant parameters. We consider a case where the coaxial connectors are of sexless type, such as APC-7 and G-900. The measurement procedure consists of two steps I and II, corresponding to the waveguide and the coaxial system, respectively. Each step requires two different standard noise sources. And one of the waveguide standards in step I is used again in step II.

First, in step I, the output port of the unknown source is changed to a waveguide system with a coax-waveguide adaptor, and the waveguide unknown is compared with the two standards. Next, in step II, the adaptor is reversely connected to the waveguide standard to change its output to the coaxial system. The coaxial unknown is then compared with these two standards in similar fashion.

The radiometer output indication is expressed as [2], [3]

$$J_i = G\left[\left(1-|\Gamma_{ri}|^2\right)T_i + |\Gamma_{ri}|^2 T_r + 2\,\mathrm{Re}\{\Gamma_{ri}T_m\} + T_n\right] \quad (3)$$

with

$$\Gamma_{ri} = \frac{\Gamma_i - \Gamma_r^*}{1-\Gamma_i\Gamma_r} \quad (4)$$

where the subscript i designates each of the sources $w1$, $w2$, xw, $w1c$, c, and x; Γi are Γr are the reflection coefficients of a noise source and a radiometer, respectively; G is the conversion coefficient of noise temperature to output indication; T_r, T_m, and T_n are, respectively, the effective temperature when looking into the radiometer at its input port, i.e., the radiometer noise temperature, the complex correlation coefficient of the radiometer expressing the degree of correlation between the direct and reflected wave components of noises generated within the front end, and the effective temperature due to noises generated within the radiometer.

This indication can be rewritten as

$$J_i = G\left[K_{ri}(T_i - T_r) + T_{n'}\right] \quad (5)$$

with

$$K_{ri} = 1 - |\Gamma_{ri}|^2 \quad (6)$$

$$T_{n'} = T_r + 2\,\mathrm{Re}\{\Gamma_{ri}T_m\} + T_n \quad (7)$$

where T_r and T_m can be measured with a method that uses a sliding short [3].

Owing to the simplicity of analysis, we assume $T_{rw} = T_{rc} = T_r$ and neglect the factor relevant to the correlation since T_m is made negligibly small by using a circulator of good isolation at the input and $|\Gamma_{ri}|$ is usually small. And let $|\Gamma_r| = 0$, i.e., $\Gamma_{ri} = \Gamma_i$ and then $K_{ri} = K_i$.

Here, we introduce the parameters R_{I} and R_{II} for the respective steps, which are directly measurable from the radiometer indications:

$$R_{\mathrm{I}} = \frac{J_{xw}^{\mathrm{I}} - J_{w1}^{\mathrm{I}}}{J_{w2}^{\mathrm{I}} - J_{w1}^{\mathrm{I}}} \quad (8)$$

$$R_{\mathrm{II}} = \frac{J_{x}^{\mathrm{II}} - J_{w1c}^{\mathrm{II}}}{J_{c}^{\mathrm{II}} - J_{w1c}^{\mathrm{II}}}. \quad (9)$$

These are rewritten by applying (5) to each indication as follows:

$$R_{\mathrm{I}} = \frac{K_{xw}(T_{xw}-T_r) - K_{w1}(T_{w1}-T_r)}{K_{w2}(T_{w2}-T_r) - K_{w1}(T_{w1}-T_r)} \quad (10)$$

$$R_{\mathrm{II}} = \frac{K_{x}(T_{x}-T_r) - K_{w1c}(T_{w1c}-T_r)}{K_{c}(T_{c}-T_r) - K_{w1c}(T_{w1c}-T_r)}. \quad (11)$$

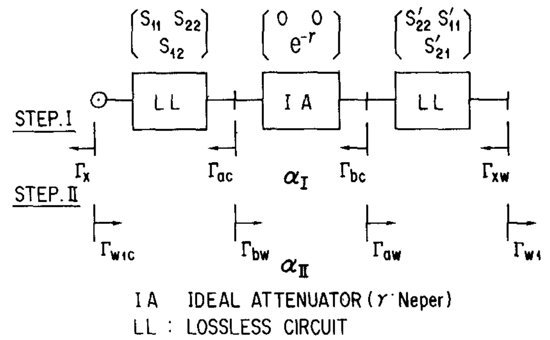

Fig. 3. Equivalent circuit of a sexless coax–waveguide adaptor expressed by the modified Wheeler network.

By applying (1) to the two noise sources with the adaptor, the following relationships can be obtained:

$$T_{xw} - T_a = \alpha_{\mathrm{I}}(T_x - T_a) \quad (12)$$

$$T_{w1c} - T_a = \alpha_{\mathrm{II}}(T_{w1} - T_a) \quad (13)$$

where α_{I} and α_{II} are the available power ratios for the adaptor in the respective steps.

These four independent equations, (10)–(13), have five unknown parameters: T_{xw}, T_{w1c}, T_x, α_{I} and α_{II}. So we cannot derive T_x from only these four equations.

Then we introduce the following relationship for α_{I} and α_{II}:

$$\epsilon = \frac{\alpha_{\mathrm{I}}}{\alpha_{\mathrm{II}}} - 1. \quad (14)$$

The maximum and the minimum value of ϵ are approximately given by (see the Appendix)

$$\epsilon\begin{pmatrix}\max\\ \min\end{pmatrix} \simeq 4\gamma\left(|\Gamma_x| + |\Gamma_{w1c}|\right)\left(|\Gamma_x| - |\Gamma_{w1c}| \pm 2|S_{11}|\right). \quad (15)$$

The parameters in this expression correspond to the symbols shown in Fig. 3, which illustrates an equivalent circuit of the adaptor for steps I and II by the modified Wheeler network: the $|S_{11}|$ and γ can be measured by the sliding short method [4]. The magnitude of ϵ is represented by the mean value from the maximum and the minimum value, i.e.,

$$\bar{\epsilon} \simeq 4\gamma\left(|\Gamma_x|^2 - |\Gamma_{w1c}|^2\right). \quad (16)$$

Using (10)–(14), we can derive

$$T_x = T_a + \frac{-(1+\epsilon)\{E(T_c - T_a) + F(T_r - T_a)\} + \sqrt{H}}{2(1+\epsilon)D} \quad (17)$$

with

$$H = (1+\epsilon)^2\{E(T_c - T_a) + F(T_r - T_a)\}^2 + 4(1+\epsilon)D(T_{w1} - T_a)\{A(T_{w2} - T_a) + B(T_{w1} - T_a) + C(T_r - T_a)\} \quad (18)$$

where

$$A=\frac{K_{w2}}{K_{xw}}R_{\mathrm{I}} \tag{19}$$

$$B=\frac{K_{w1}}{K_{xw}}(1-R_{\mathrm{I}}) \tag{20}$$

$$C=1-A-B \tag{21}$$

$$D=\frac{K_x}{K_{w1c}}\frac{1}{1-R_{\mathrm{II}}} \tag{22}$$

$$E=-\frac{K_c}{K_{w1c}}\frac{R_{\mathrm{II}}}{1-R_{\mathrm{II}}} \tag{23}$$

and

$$F=1-D-E. \tag{24}$$

Equation (17) is complicated. But if T_{w2}, T_c, and $T_r=T_a$, it can be simplified as follows:

$$T_x=T_a+\sqrt{\frac{1}{1+\epsilon}\frac{K_{w1}K_{w1c}}{K_{xw}K_x}(1-R_I)(1-R_{\mathrm{II}})}\,(T_{w1}-T_a). \tag{25}$$

At the same time, α_{I} is given by

$$\alpha_{\mathrm{I}}=\sqrt{(1+\epsilon)\frac{K_{w1}K_x}{K_{xw}K_{w1c}}\frac{1-R_{\mathrm{I}}}{1-R_{\mathrm{II}}}}. \tag{26}$$

That is, we must use a noise source of the room-temperature type as one of the two standards in each step for $T_{w2}=T_c=T_a$ and a radiometer whose front stage has an isolator at room temperature for $T_r=T_a$.

In the case where the connector system is of the sexual type, e.g., the output connector of the unknown is of the female type, the measurement procedure consists of three steps, as shown in Fig. 4: step I uses a coax(male)-waveguide adaptor for the unknown; step II uses the same adaptor placed reversely in front of the waveguide standard (NS.1). At the same time, two coax (male–male) adaptors are placed in front of the coaxial standard and the unknown, respectively, and step III uses a coax (female–female) adaptor in front of the waveguide standard changed to the coaxial type. Here, we assume that the adaptors of coax (male–male) and (female–female) are almost the same in electrical characteristics. If T_{w2}, T_c, T_{cm}, and $T_r=T_a$, T_x can be expressed as

$$T_x=T_a+\frac{K_{w1}}{K_{xw}}(1-R_{\mathrm{I}})(T_{w1}-T_a)\cdot\frac{1}{1+\mu}\sqrt{\frac{K_{w1m}K_x}{K_{xm}K_{w1f}}\frac{1-R_{\mathrm{II}}}{1-R_{\mathrm{III}}}\frac{1}{1+\nu}} \tag{27}$$

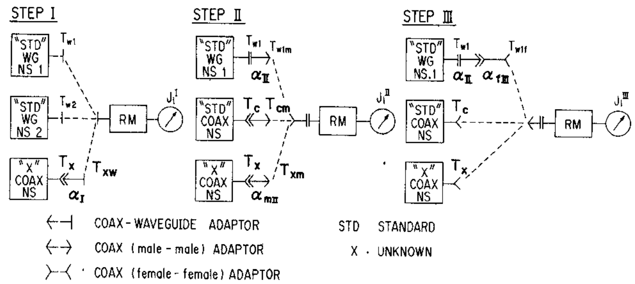

Fig. 4. Schematic diagram depicting the measurement procedure and system for a coaxial connector system of the sexual type.

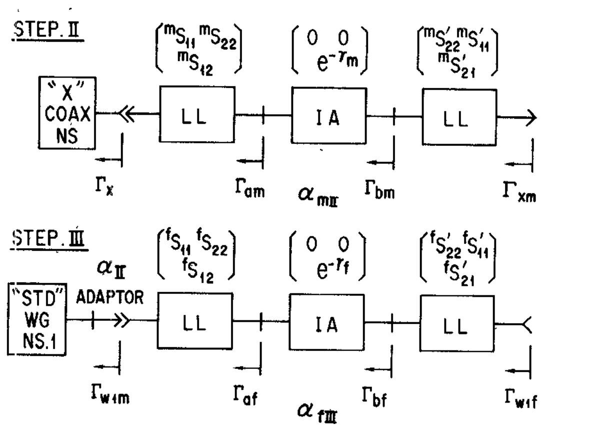

Fig. 5. Equivalent circuits of two coaxial adaptors (male–male and female–female) used for the sexual coaxial connector system.

where

$$R_{\mathrm{I}}=\frac{J_{xw}^{\mathrm{I}}-J_{w1}^{\mathrm{I}}}{J_{w2}^{\mathrm{I}}-J_{w1}^{\mathrm{I}}} \tag{28}$$

$$R_{\mathrm{II}}=\frac{J_{xm}^{\mathrm{II}}-J_{w1m}^{\mathrm{II}}}{J_{cm}^{\mathrm{II}}-J_{w1m}^{\mathrm{II}}} \tag{29}$$

$$R_{\mathrm{III}}=\frac{J_{x}^{\mathrm{III}}-J_{w1f}^{\mathrm{III}}}{J_{c}^{\mathrm{III}}-J_{w1f}^{\mathrm{III}}} \tag{30}$$

$$\mu=\frac{\alpha_{\mathrm{I}}}{\alpha_{\mathrm{II}}}-1 \tag{31}$$

and

$$\nu=\frac{\alpha_{m\mathrm{II}}}{\alpha_{f\mathrm{III}}}-1. \tag{32}$$

The maximum and the minimum values for μ and ν are, respectively, given by

$$\mu\binom{\max}{\min}\simeq 4\gamma(|\Gamma_x|+|\Gamma_{xw}|)(|\Gamma_x|-|\Gamma_{xw}|\pm 2|S_{11}|) \tag{33}$$

$$\nu\binom{\max}{\min}\simeq 4\gamma_m\left\{\left(|{}^mS_{11}|^2-|{}^fS_{11}|^2\right)+\left(|\Gamma_x|^2-|\Gamma_{w1m}|^2\right)\pm 2\left(|{}^mS_{11}||\Gamma_x|+|{}^fS_{11}||\Gamma_{w1m}|\right)\right\}+4|\gamma_m-\gamma_f||\Gamma_{af}|^2. \tag{34}$$

The parameters associated to the expressions correspond to the symbols shown in Figs. 4 and 5, which illustrate the equivalent circuits of the coaxial adaptors (male–male and female–female) in steps II and III.

The second term in (34) may be neglected since usually $|\Gamma_{aj}| \ll 1$ and $\gamma_m \simeq \gamma_f$.

III. Error Analysis

We consider the major sources of error to temperature calibration by this method when the coaxial connector is of the sexless type.

1) Error Due to Approximation of ϵ to $\bar{\epsilon}$: The maximum value of the error is

$$|\Delta\epsilon| \simeq 8\gamma |S_{11}|(|\Gamma_x| + |\Gamma_{w1c}|). \tag{35}$$

Its contribution to the calibration error when $\epsilon \ll 1$ is given as

$$|\Delta T_x|_\epsilon \simeq \frac{1}{2}|T_x - T_a|\,|\Delta\epsilon|. \tag{36}$$

Fig. 6 shows an example of the calculation results expressing the relationships of $|\Delta\epsilon|$ and $|\Delta T_x|$ to γ in the case where the unknown source is of the liquid nitrogen cooled type.

2) Error Due to Deviation of T_{w2}, T_c and T_r from T_a: Their contributions to the calibration error are

$$|\Delta T_x|_{w2,a} \simeq \left|\frac{A(T_{w1} - T_a)}{2D(T_x - T_a) + E(T_c - T_a)}\right| |T_{w2} - T_a| \tag{37}$$

$$|\Delta T_x|_{c,a} \simeq \left|\frac{E(T_x - T_a)}{2D(T_x - T_a) + E(T_c - T_a)}\right| |T_c - T_a| \tag{38}$$

and

$$|\Delta T_x|_{r,a} \simeq \left|\frac{C(T_{w1} - T_a) - F(T_x - T_a)}{2D(T_x - T_a) + E(T_c - T_a)}\right| |T_r - T_a| \tag{39}$$

where A, C, D, E, and F correspond to the quantities in (19), (21), (22), (23), and (24), respectively, and the magnitudes of C and F are considered much less than unity.

3) Error Due to Neglect of $|\Gamma r|$: The maximum difference between $|\Gamma_{ri}|^2$ and $|\Gamma_i|^2$ for each of the K's in (25) is

$$\left| |\Gamma_{ri}|^2 - |\Gamma_i|^2 \right| \simeq |\Gamma_r|(1 - |\Gamma_i|^2)\left| |\Gamma_r| - 2|\Gamma_i| \right|. \tag{40}$$

Its contribution to the calibration error is

$$|\Delta T_x|_{ri,i} \simeq \frac{1}{2}|T_x - T_a|\left| |\Gamma_{ri}|^2 - |\Gamma_i|^2 \right|. \tag{41}$$

For example, if $T_x = 80K$, $|\Gamma_r| = 0.01$, and $|\Gamma_i| = 0.1$, the contribution to the calibration error due to each of the K's is about $0.21K$, so this may become one of significant errors. Therefore, the magnitudes of the reflection coefficients should be as small as possible.

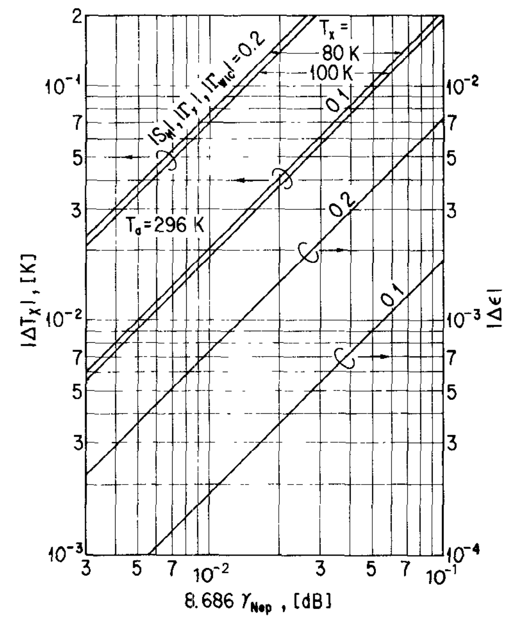

Fig. 6. Error due to the approximation of ϵ to $\bar{\epsilon}$ and the calibration error of the unknown versus γ.

4) Measurement Errors of R_I and R_{II}: Their contributions are written as

$$|\Delta T_x|_{R_{\rm I}} \simeq \frac{1}{2}\left|\frac{T_x - T_a}{1 - R_{\rm I}}\right| |\Delta R_{\rm I}| \tag{42}$$

$$|\Delta T_x|_{R_{\rm II}} \simeq \frac{1}{2}\left|\frac{T_x - T_a}{1 - R_{\rm II}}\right| |\Delta R_{\rm II}|. \tag{43}$$

5) Error Due to Approximation of $T_{rw} = T_{rc}$: Using (10) and (11), we can have

$$|\Delta R_{\rm I}|_{rw,c} \simeq |R_{\rm I}|\left(\left|\frac{|\Gamma_{xw}|^2 - |\Gamma_{w1}|^2}{T_{xw} - T_{w1}}\right| + \left|\frac{|\Gamma_{w2}|^2 - |\Gamma_{w1}|^2}{T_{w2} - T_{w1}}\right|\right) \cdot |T_{rw} - T_c| \tag{44}$$

$$|\Delta R_{\rm II}|_{rw,c} \simeq |R_{\rm II}|\left(\left|\frac{|\Gamma_x|^2 - |\Gamma_{w1}|^2}{T_x - T_{w1}}\right| + \left|\frac{|\Gamma_c|^2 - |\Gamma_{w1c}|^2}{T_c - T_{w1c}}\right|\right) \cdot |T_{rw} - T_c|. \tag{45}$$

Inserting these into (42) and (43), respectively, we can know their contributions to the calibration error.

6) Error Due to Neglect of Factors Relevant to the Correlation: In fact, (10) and (11) should have the factors relevant to the correlation; i.e., the former has $2\,\mathrm{Re}\{\Gamma_{xw}T_m\} - 2\,\mathrm{Re}\{\Gamma_{w1}T_m\}$ in the numerator and $2\,\mathrm{Re}\{\Gamma_{w2}T_m\} - 2\,\mathrm{Re}\{\Gamma_{w1}T_m\}$ in the denominator, and the latter $2\,\mathrm{Re}\{\Gamma_x T_m\} - 2\,\mathrm{Re}\{\Gamma_{w1c}T_m\}$ in the numerator and $2\,\mathrm{Re}\{\Gamma_c T_m\} - 2\,\mathrm{Re}\{\Gamma_{w1c}T_m\}$ in the denominator. Then we have

$$|\Delta R_{\rm I}|_m \simeq 2|R_{\rm I}|\left(\frac{|\Gamma_{xw}| + |\Gamma_{w1}|}{|T_{xw} - T_{w1}|} + \frac{|\Gamma_{w2}| + |\Gamma_{w1}|}{|T_{w2} - T_{w1}|}\right)T_m \tag{46}$$

$$|\Delta R_{\rm II}|_m \simeq 2|R_{\rm II}|\left(\frac{|\Gamma_x| + |\Gamma_{w1}|}{|T_x - T_{w1}|} + \frac{|\Gamma_c| + |\Gamma_{w1c}|}{|T_c - T_{w1c}|}\right)T_m. \tag{47}$$

Inserting these into (42) and (43), respectively, we can know their contributions to the calibration error.

7) Errors Due to Connector Insertion Loss and its Reproducibility: Much consideration should be taken to reduce them since the errors may possibly become dominant in coaxial system, and their effect is more pronounced as the difference between the noise temperature of a noise source and the room temperature become larger.

IV. Conclusions

A practical method has been given that is capable of easily calibrating a coaxial noise source with a waveguide standard without directly evaluating the electrical characteristics of the coax-waveguide adaptor. It has been shown that the measurement equation can be simplified when one of the standards is of the room-temperature type and the effective temperature of the radiometer when looking inside at the input port is equal to the room temperature. It has also been shown that the errors due to the deviation of these temperatures from the room temperature are less susceptible to influence as the noise temperature of the waveguide standard approaches that of the unknown.

Appendix

The quantities $\alpha_{\rm I}$ and $\alpha_{\rm II}$ shown in Fig. 3 are given by

$$\alpha_{\rm I} = \frac{e^{-2\gamma}\left(1-|\Gamma_{ac}|^2\right)}{1-e^{-4\gamma}|\Gamma_{ac}|^2} \tag{A1}$$

$$\alpha_{\rm II} = \frac{e^{-2\gamma}\left(1-|\Gamma_{aw}|^2\right)}{1-e^{-4\gamma}|\Gamma_{aw}|^2} \tag{A2}$$

where $\alpha_{\rm I}$ and $\alpha_{\rm II}$ are monotonically decreasing functions of $|\Gamma_{ac}|$ and $|\Gamma_{aw}|$, respectively.

By adopting the lossless condition to the LL circuits, $|\Gamma_{ac}|$ and $|\Gamma_{aw}|$ are given by

$$|\Gamma_{ac}| = \left|\frac{S_{11}^* - \Gamma_x}{1 - S_{11}\Gamma_x}\right| \tag{A3}$$

and

$$|\Gamma_{aw}| = \left|\frac{S_{11} - \Gamma_{w1c}}{1 - S_{11}^*\Gamma_{w1c}}\right|. \tag{A4}$$

By using these equations and (14) and the condition $\gamma \ll 1$, (15) can be derived approximately.

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